Section 4.5

Optimization Problems

Optimization problems are solved similarly to related-rates problems. It is important to notice both the similarities and the differences.

Procedure 4.5.1 (Solving Optimization Problems)**.**

1. Find what you know and don't know.
   (a) Sketch a geometric picture to represent the problem.
   (b) Write down the quantity you need to minimize/maximize and all variables needed to compute this quantity.
2. Write the objective function, a function giving the desired optimized quantity in terms of given variables.
3. Identify constraint equations.
4. Use constraint to eliminate all but one independent variable from the objective function.
5. Use Section 4.1 to find the absolute min/max as needed. Remember to check endpoints.

Ladder over a fence. An 8 ft fence runs parallel to the side of a house 5 ft away. What is the length of the shortest ladder that clears the fence and reaches the house? Assume the house is 20 ft tall and there is 20 ft of ground past the fence.

Optimal Flower Garden. A rectangular flower garden with an area of 30 m^2 is surrounded by a grass border 1 m wide on two opposite sides and 2 m wide on the others. What dimensions of the garden minimize the combined area of the garden and borders?

Cones. A right circular cone is inscribed inside a larger right circular cone with a volume of 120 cm^3. The axes of the cones coincide, and the vertex of the inner cone touches the center of the base of the outer cone. Find the ratio of the heights of the cones that maximize the volume of the inner cone.

Quickest Boat Trip. A boat on the ocean is 4 mi from the nearest point on a straight shoreline; that point is 12 mi fro a restaurant on the shore. A woman plans to row the boat straight to a point on the shore and then walk along the shore to the restaurant. If she walks 3 mi/hr and rows 2 mi/hr, at what point on the shore should she land to minimize the total travel time?